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Skewed parton distributions for $B \to \pi$ transitions

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Abstract

We investigate the b-u skewed parton distributions (SPDs) for $B \to \pi$ transitions and determine the contributions from several sources (overlaps of soft light-cone wave functions, quark-antiquark annihilations and meson resonances). The $B \to \pi$ transition form factors, which are relevant in exclusive semi-leptonic and non-leptonic B-decays, are obtained by integrating the b-u SPDs over the momentum fraction x. A phenomenological determination of the relevant parameters allows us to predict the form factors and to obtain the branching ratios for semi-leptonic $B \to \pi$ decays.

1 Introduction

A good theoretical understanding of heavy-to-light meson form factors, which encode the confinement of the quarks in the hadronic bound states, are of utmost interest. Accurate predictions of the form factors would permit the determination of the less well-known Cabbibo-Kobayashi-Maskawa (CKM) matrix elements from experimental rates of exclusive heavy meson decays. For instance, in the case of the semi-leptonic $B \to \pi$ transitions, on which we focus our interest in this article, the relevant entry in the CKM matrix is $|V_{\rm ub}|$. Its present value is 0.0035 with an uncertainty of about 0.001 [1, 2]. The form factors for transitions from the B meson to light mesons also form an important ingredient of the calculation of exclusive non-leptonic B decays, e.g. for $B \to \pi\pi$. Thus, not surprisingly, the heavy-to-light form factors attracted the attention of theoreticians, and many articles have been devoted to their investigation. The theoretical approaches utilised in these articles reach from the quark model [3], overlaps of light-cone wave functions [4, 5], perturbative QCD [6, 7], the heavy quark symmetries [8, 9] and QCD sum rules [10, 11], to name a few.

In several of these approaches there are two distinct and prominent dynamical mechanisms: The $B\pi$ resonances which control the form factors at small recoil and the overlap of meson wave functions which dominates at large recoil. Other mechanisms, like the perturbative one, provide only small corrections. The crucial problem arises then, how to match these two contributions at intermediate recoil. In this article we are proposing a new approach which is based on the concept of generalised or, as frequently termed, skewed parton distributions which has recently been invented in the context of deeply virtual Compton scattering [12]. The SPDs are defined as non-forward matrix elements of non-local currents. They are hybrid objects in this respect which share the properties of ordinary parton distributions and form factors. We are going to introduce b-u SPDs as a parametrisation of the soft $B \to \pi$ matrix element. The chief advantage of the SPDs for $B \to \pi$ transitions is that they clearly separate resonance and overlap contribution and thus allow the superposition of both the contributions in an unambiguous way. This SPD approach may constitute an important step forward towards a unified description of the $B \to \pi$ form factors at small and large recoil, although we are aware that there is still a number of open questions to be answered before a satisfactory and complete description of the $B \to \pi$ transition form factors has been achieved.
Our paper is organised as follows: In Sect. 2 we present the basic definitions and the kinematics. In Sect. 3 we introduce the b-u SPDs and discuss contributions to them from various sources. From the SPDs we calculate the $B \to \pi$ form factors as functions of the momentum transfer, q^2. The results are presented in Sect. 4 together with a comparison to other results, an assessment of their theoretical uncertainties and an evaluation of the semi-leptonic $B \to \pi$ decay rates. In this section we also check our form factors against the unitarity bounds derived in Ref. [13]. The summary is presented in Sect. 5.

2 Kinematics

To be specific we consider the semi-leptonic decay $\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}_l$; all our results can straightforwardly be adapted to other $B \to \pi$ transitions. The form factors for $\bar{B}^0 \to \pi^+$ transitions are frequently defined by (see e.g. [4, 10, 13])

$$\begin{aligned} \langle \pi^+; p' | \bar{\mathrm{u}}(0) \gamma_\mu \mathrm{b}(0) | \bar{B}^0; p \rangle &= F_+(q^2) \left(p_\mu + p'_\mu - \frac{M_B^2 - M_\pi^2}{q^2}\, q_\mu \right) \\ &+ F_0(q^2)\, \frac{M_B^2 - M_\pi^2}{q^2}\, q_\mu \,, \end{aligned} \tag{1}$$

where $q = p - p'$ and M_B (M_π) is the B (π) mass. The form factors defined in (1) are subject to the kinematical constraint $F_+(0) = F_0(0)$. For our purpose of investigating the SPDs for $B \to \pi$ transitions it is more convenient to use the alternative covariant decomposition

$$\langle \pi^+; p' | \bar{\mathrm{u}}(0) \gamma_\mu \mathrm{b}(0) | \bar{B}^0; p \rangle = F^{(1)}(q^2)\, p'_\mu + F^{(2)}(q^2) \left(q_\mu - \frac{q^2}{M_B^2} p_\mu \right) . \tag{2}$$

The two sets of form factors are related by

$$
\begin{aligned}
F_{+} &= \frac{1}{2}\left(F^{(1)} - \frac{q^2}{M_B^2}F^{(2)}\right) \,, \\
F_{0} &= \frac{1}{2}\left(1 - \frac{q^2}{M_B^2 - M_\pi^2}\right) F^{(1)} + \frac{q^2}{2M_B^2}\frac{M_B^2 + M_\pi^2}{M_B^2 - M_\pi^2}\left(1 - \frac{q^2}{M_B^2 + M_\pi^2}\right) F^{(2)} \,.
\end{aligned}
\tag{3}
$$

At $q^2 = 0$ the form factors F_+ and F_0 are solely determined by the form factor $F^{(1)}$. Most convenient for the calculation of the $B \to \pi$ transition form factors in terms of SPDs is a frame of reference where the hadron momenta are collinear to each other; this frame may be viewed as a generalisation of a Breit frame. We introduce light-cone coordinates $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ and $\mathbf{v}_\perp = (v^1, v^2)$ for any four-vector v and use component notation $v = [v^+, v^-, \mathbf{v}_\perp]$. Defining the so-called skewedness parameter by

$$
\zeta = \frac{q^+}{p^+} = 1 - \frac{p'^+}{p^+} \,, \tag{4}
$$

we can write the B and π momenta in our frame of reference as

$$
p = \left[p^+ \,,\, \frac{M_B^2}{2p^+} \,,\, \mathbf{0}_\perp \right] , \qquad p' = \left[(1-\zeta)p^+ \,,\, \frac{M_\pi^2}{2p^+\,(1-\zeta)} \,,\, \mathbf{0}_\perp \right] . \tag{5}
$$

Positivity of the energy of the final state meson implies $\zeta < 1$. The momentum transfer is given by

$$
q^2 = \zeta M_B^2 \left(1 - \frac{M_\pi^2}{M_B^2(1-\zeta)}\right) . \tag{6}
$$

The skewedness parameter ζ covers the interval $[0, 1 - M_\pi/M_B]$ in parallel with the variation of the momentum transfer from zero (we neglect the lepton mass here) to $q^2_{\max} = (M_B - M_\pi)^2$ in the physical region of the $B \to \pi$ transitions. In contrast to the case of form factors in the space-like region [14], there is no frame for $B \to \pi$ transitions in which the skewedness parameter can be chosen to be zero. In the following we will neglect the pion mass in the calculation of the SPDs and form factors.

For convenience we quote the light-cone components of the current matrix element (2) in the frame of reference (5):

$$
\begin{aligned}
\langle \pi^+; p' | \bar{\mathrm{u}}(0)\gamma^+ \mathrm{b}(0) | \bar{B}^0; p \rangle &= F^{(1)}(q^2)\,(1 - \frac{q^2}{M_B^2})\,p^+ \,, \\
\langle \pi^+; p' | \bar{\mathrm{u}}(0)\gamma^- \mathrm{b}(0) | \bar{B}^0; p \rangle &= F^{(2)}(q^2)\,(1 - \frac{q^2}{M_B^2})\,\frac{M_B^2}{2p^+} \,.
\end{aligned}
\tag{7}
$$

The matrix elements of the transverse currents are zero.

3 b-u skewed parton distributions

We define the b-u SPD $\tilde{\mathcal{F}}_\zeta^{(1)}$ by the non-forward matrix elements

$$\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixp^+z^-}\, \langle \pi^+; p'|\bar{\mathrm{u}}(0)\gamma^+\mathrm{b}(z^-)|\bar{B}^0; p\rangle = (1-\zeta)\, \tilde{\mathcal{F}}_\zeta^{(1)}(x,q^2)\,, \tag{8}$$

where $x = k^+/p^+$ is the fraction of plus-components of the b-quark and B-meson momenta. The second SPD, $\tilde{\mathcal{F}}_\zeta^{(2)}$, is analogously defined with γ^+ being replaced by γ^-, see also Eq. (7). In the frame of reference chosen by us the momentum transfer and the skewedness parameter are related to each other by Eq. (6), $q^2 = \zeta M_B^2$. This relation makes the q^2 variable in $\widetilde{\mathcal{F}}_\zeta^{(i)}$ redundant. For the ease of notation we will, therefore, omit it in the following.

Depending on the value of x, the SPDs describe different physical situations [12]: For $1 \geq x \geq \zeta$ a b quark with momentum fraction x is taken out of the B meson and a u quark carrying a momentum fraction $x' = k^{+\prime}/p^{+\prime}$ (with respect to the final state meson) is inserted back, turning the B meson into a pion (see Fig. 1a)[1]. This part of the SPDs will be modelled as overlaps of B and π light-cone wave functions. For $0 \leq x < \zeta$ the B meson emits a b$\bar{\mathrm{u}}$ pair and the remaining partons form the pion (see Fig. 1b). According to Brodsky and Hwang [15] this contribution can be described by non-diagonal light-cone wave function overlaps for $n+2 \to n$ parton processes [2]. In addition, as pointed out by Radyushkin [16], $B\pi$ resonances contribute to the SPDs in that region (see Fig. 1c). These considerations lead to the following decomposition of the b-u SPDs in the interval $0 \leq x \leq 1$

$$\widetilde{\mathcal{F}}_\zeta^{(i)}(x) = \theta(x-\zeta)\widetilde{\mathcal{F}}_{\zeta\,\mathrm{ove}}^{(i)}(x) \,+\, \theta(\zeta - x)\left[\widetilde{\mathcal{F}}_{\zeta\,\mathrm{ann}}^{(i)}(x) \,+\, \widetilde{\mathcal{F}}_{\zeta\,\mathrm{res}}^{(i)}(x)\right]\,, \tag{10}$$

where the three parts of the SPDs labelled ove, ann and res refer to the contributions from Fig. 1 a), b) and c), respectively. The relative importance of the overlap contribution to the SPDs on the one side and the sum of annihilation and resonance one on the other side, change with the momentum transfer as a consequence of the relation (6). At large recoil, $q^2 \simeq 0$, the annihilation and resonance parts do not contribute while they dominate at small recoil, $q^2 \simeq q^2_{\max}$. We stress that the superposition (10) is controlled by the momentum transfer or the skewedness parameter ζ in an unambiguous way, i.e. there is no danger of double counting. The b-u SPDs exist in a third region of the variable x, namely for $-1+\zeta \leq x < 0$ where they describe the situation that a b quark with a negative momentum fraction is emitted from the B meson and u quark is absorbed. Re-interpreting a quark with a negative momentum fraction as an antiquark with a positive fraction, one

[1] The momenta of the active b and u quarks read in our frame of reference

$$k = \left[\, xp^+\,,\ \frac{m_\mathrm{b}^2 + k_\perp^2}{2xp^+}\,,\ \mathbf{k}_\perp \right]\,, \qquad k' = \left[\, (x-\zeta)p^+\,,\ \frac{m_\mathrm{u}^2 + k_\perp^2}{2(x-\zeta)p^+}\,,\ \mathbf{k}_\perp \right]\,. \tag{9}$$

[2] The existence of this contribution has been stressed by Sawicki [17] some time ago.

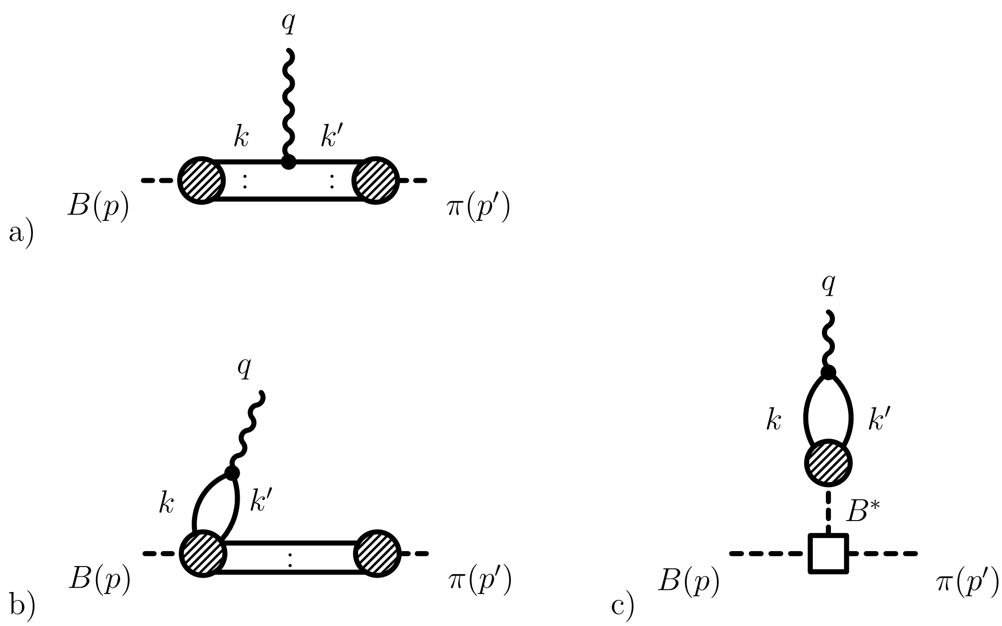

Figure 1: Overlap (a), annihilation (b) and resonance (c) contributions to $B \to \pi$ transitions. The dots indicate that any number of spectators may contribute.

finds that the region $-1+\zeta \leq x < 0$ describes the emission of a $\overline{\mathrm{b}}$-quark and the absorption of a $\overline{\mathrm{u}}$ one. This re-interpretation implies the relation

$$\widetilde{\mathcal{F}}_\zeta^{(i)\,\overline{\mathrm{b}}-\overline{\mathrm{u}}}(x) = -\widetilde{\mathcal{F}}_\zeta^{(i)\,\mathrm{b}-\mathrm{u}}(\zeta - x) \tag{11}$$

for the SPDs ($x \geq \zeta$). By way of exception we here quote the quark-flavour labels. Since the probability of finding a $\mathrm{b}\overline{\mathrm{b}}$ sea-quark pair in the B meson is practically zero, $\widetilde{\mathcal{F}}_\zeta^{(i)}(x) \simeq 0$ in the region $-1+\zeta \leq x < 0$ to a very high degree of accuracy.
By comparison of (7) and (8) one finds the reduction formula

$$F^{(i)}(q^2) \;=\; \int_0^1 \mathrm{d}x\, \widetilde{\mathcal{F}}_\zeta^{(i)}(x) \tag{12}$$

for $i = 1, 2$. The range of the x integration is restricted to the interval $[0, 1]$ since contributions from $\overline{\mathrm{b}}$ quarks or, in other words, from negative momentum fractions are absent in the form factors.
As already mentioned, we describe the overlap part of the SPDs by light-cone wave functions for the B and the π mesons. To begin with we consider the valence Fock states of the B and π mesons. The corresponding light-cone wave functions, Ψ_B and Ψ_π, respectively,

provide the overlap contribution

$$\widetilde{\mathcal{F}}^{(1)}_{\zeta\,\mathrm{ove}}(x) \;=\; \frac{2}{1-\zeta}\int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\;\Psi^*_\pi(x'=\frac{x-\zeta}{1-\zeta},\mathbf{k}_\perp)\;\Psi_B(x,\mathbf{k}_\perp)\,, \tag{13}$$

where $\mathbf{k}_\perp$ is the intrinsic transverse momentum of the b (u) quark with respect to the B (π)-meson momentum. As a consequence of the collinearity of the two meson momenta in our frame of reference the transverse momentum in the argument of the π wave function is the same as in the B wave function, while the longitudinal momentum fraction is shifted. For the pion valence Fock state wave function we take a simple Gaussian ansatz

$$\Psi_\pi(x,\mathbf{k}_\perp) = \frac{\sqrt{6}}{f_\pi}\,\exp\left[-\frac{1}{8\pi^2 f_\pi^2}\,\frac{\mathbf{k}_\perp{}^2}{x\,(1-x)}\right] \tag{14}$$

with the associated asymptotic distribution amplitude

$$\phi_\pi^{\mathrm{AS}}(x) = 6x(1-x)\,. \tag{15}$$

f_π (=132 MeV) is the usual pion decay constant. The pion's transverse size parameter is fixed by the chiral anomaly to $(2\sqrt{2}\pi f_\pi)^{-1}$ [18]. The wave function (14) being normalised to 0.25 at a scale of 1 GeV, has been tested against experiment and found to work satisfactorily in many hard exclusive reactions involving pions (cf. [19] for instance). It is also supported by recent QCD sum rule results (cf. [20] for instance), by a study of power corrections [21] and by the instanton model [22].
For the b$\overline{\mathrm{q}}$ wave function of the B meson we use a slightly modified version of the Bauer-Stech-Wirbel (BSW) function [4] which has been shown to be useful in weak decays [3]

$$\Psi_B(x,\mathbf{k}_\perp) = \frac{f_B}{2\sqrt{6}}\,\phi_B(x)16\pi^2 a_B^2\;\exp\left[-a_B^2\mathbf{k}_\perp{}^2\right], \tag{16}$$

where the distribution amplitude is given by

$$\phi_B(x) = \mathcal{N}\,x\,(1-x)\;\exp\left[-a_B^2\,M_B^2\;(x-x_0))^2\right]\,. \tag{17}$$

The distribution amplitude ϕ_B exhibits a pronounced peak, its position is approximately at $x \simeq x_0 = m_\mathrm{b}/M_B$. For a b-quark mass, m_b, of 4.8 GeV the value of x_0 is 0.91. This property of the B-meson distribution amplitude parallels the theoretically expected and experimentally confirmed behaviour of heavy meson fragmentation functions. The constant $\mathcal{N}$ in Eq. (17) is fixed by the condition

$$\int_0^1 \phi_B(x)\,dx = 1\,. \tag{18}$$

For the B-meson decay constant f_B we take a value of 180 MeV which is supported by recent lattice gauge theory analyses [24]. The only remaining free parameter in the B-meson

[3] A Gaussian as in (14) for the B meson has theoretical deficiencies in the formal limit $M_B \to \infty$ and is, therefore, in conflict with the heavy quark effective theory [23].

wave functions (16) is the transverse size parameter, a_B, which we fix by normalising the B-meson's valence Fock state probability to unity. This leads to a value of 1.51 GeV^{-1} for a_B if a value of 4.8 GeV is chosen for the b-quark (pole) mass [25]. The parameter $\bar{\Lambda}$, given by the B-meson and b-quark mass difference, acquires a value of 480 MeV. The constant $\mathcal{N}$ in Eq. (17) then takes a value of 54.7. The maximum of the distribution amplitude $\phi_B(x)$ is located at $x_{\max} = 0.86$. We checked that our final results only mildly depend on variation of the parameters m_b and f_B and of the probability of the B meson's valence Fock state.
Performing the trivial $\mathbf{k}_\perp$ integration in (13), we find

$$\widetilde{\mathcal{F}}^1_{\zeta\,\text{ove}}(x) = 8\pi^2 f_B f_\pi\, a_B^2\, \frac{(x-\zeta)\,(1-x)}{8\pi^2 f_\pi^2 a_B^2\,(x-\zeta)\,(1-x)\,+\,(1-\zeta)^2}\,\frac{\phi_B(x)}{1-\zeta}\,. \tag{19}$$

We see that $\widetilde{\mathcal{F}}^1_{\zeta\,\text{ove}}(x) \propto (x-\zeta)$ for $x \to \zeta$ and ζ fixed. In the formal limit $M_B \to \infty$, this SPD behaves as $M_B^{-3/2}$.
Of course, the result (19) can easily be translated to other choices of the pion and B-meson wave functions. The numerical results will not change significantly as long as distribution amplitudes are used which are close to the asymptotic one in case of the pion and strongly peaked at large x in case of the B meson.
From the wave functions (14) and (16) one may also calculate the overlap part of the SPD $\widetilde{\mathcal{F}}^2_{\zeta\,\text{ove}}$ in full analogy to (13). However, due to additional $k_\perp$ factors, arising from the matrix elements of the γ^- current between the light-cone helicity spinors, $\widetilde{\mathcal{F}}^2_{\zeta\,\text{ove}}$ is power-suppressed to order $\bar{\Lambda}/M_B$ at least as compared to $\widetilde{\mathcal{F}}^1_{\zeta\,\text{ove}}$ and, hence, neglected.
In principle, the overlap parts of the SPDs receive contributions from all Fock states. The generalisation of the overlap representation (13) to higher Fock states is a straightforward application of the methods outlined in [14]. Using suitably generalised wave functions for the higher Fock states (cf. [14]), one can show that the higher Fock state contributions to $\widetilde{\mathcal{F}}^1_{\zeta\,\text{ove}}$ are very small and can be neglected. It is not only the tiny probabilities of the higher B-meson Fock states which is responsible for this fact. Even more important for the suppression of these contributions is a conspiracy of the factor $(1-x)^{n(N)}$ appearing in the N-particle Fock state contribution to the SPD $\widetilde{\mathcal{F}}^1_{\zeta\,\text{ove}}$ and the strongly peaked shape of the B-meson wave function. Here, $n(N)$ is a positive integer increasing with N [14]. Since $x_0 = 1 - \bar{\Lambda}/M_B$ one may regard the contribution of the N-particle Fock state as a power correction $(\bar{\Lambda}/M_B)^{n(N)}$ to (19). Thus, to a high degree of accuracy, the restriction to the valence contribution suffices for the overlap part of the b-u SPDs.
In order to estimate the annihilation part of the SPDs we can restrict ourselves again to the parton process with the minimal number of partons participating, namely the process $\mathrm{b}\bar{\mathrm{d}}\mathrm{u}\bar{\mathrm{u}} \to \bar{\mathrm{d}}u$, and we are going to show that this contribution is negligibly small, too. Numbering the b quark by 1 and $\bar{\mathrm{u}}$ by 4 and noting that the momentum q is shared by the b and the $\bar{\mathrm{u}}$ quark, one finds the conditions $x_4 = \zeta - x_1$ and $\mathbf{k}_{\perp 4} = -\mathbf{k}_{\perp 1}$ in our frame of reference defined by Eq. (5). In combination with momentum conservation this leads to the relations $x_3 = 1 - \zeta - x_2$ and $\mathbf{k}_{\perp 3} = -\mathbf{k}_{\perp 2}$ for the momentum fractions and transverse momenta of the additional $\bar{\mathrm{d}}$ and u quarks. With these results in mind one arrives at the

following overlap contribution [15]

$$\widetilde{\mathcal{F}}^{1}_{\zeta\,\mathrm{ann}}(x) = \frac{2}{1-\zeta}\int_0^{1-\zeta} \mathrm{d}x_2 \int \frac{\mathrm{d}^2\mathbf{k}_{\perp 1}\mathrm{d}^2\mathbf{k}_{\perp 2}}{(16\pi^2)^3}\,\Psi^*_\pi(x_2' = \frac{x_2}{1-\zeta}, \mathbf{k}_{\perp 2})\,\Psi_{B,4}(x_i, \mathbf{k}_{\perp i})\,, \tag{20}$$

where Ψ_π is the pion valence Fock state wave function (14) and $\Psi_{B,4}$ the four particle wave function of the B meson. Generalising the wave function (16), (17) in a straightforward fashion to the four-particle case, we find

$$\widetilde{\mathcal{F}}^{(1)}_{\zeta\,\mathrm{ann}}(x) \propto (1-\zeta)^2(\zeta - x)\exp\left[-a_B^2 M_B^2 (x-x_0)^2\right], \tag{21}$$

i.e. the annihilation contribution to $\widetilde{\mathcal{F}}^{(1)}_{\zeta}$ is exponentially damped except for $\zeta \gtrsim x_0$ (see Eq. (10)) and $x \simeq x_0$. This region, however, is suppressed by the factors $(1-\zeta)^2$ and $\zeta - x$. Thus, the annihilation contribution is very small and can safely be neglected. Similar arguments hold for $\widetilde{\mathcal{F}}^{(2)}_{\zeta\,\mathrm{ann}}$.

For the resonance contribution (see Fig. 1c) we concentrate on the resonance that is closest to the physical decay region, i.e. on the B^{*-} vector meson. From the Lorentz structure of the $BB^*\pi$ vertex [9] we infer

$$\begin{aligned} \widetilde{\mathcal{F}}^{(1)}_{\zeta\,\mathrm{res}}(x) &= \frac{f_{B^*}\,g_{BB^*\pi}}{M_{B^*}}\left(M^2_{B^*} - \frac{1}{2}\zeta M_B^2\right)\frac{\phi_{B^*}(x/\zeta)}{M^2_{B^*} - \zeta\,M_B^2}\,, \\ \widetilde{\mathcal{F}}^{(2)}_{\zeta\,\mathrm{res}}(x) &= -\frac{1}{2}\,M_B^2\,\frac{f_{B^*}\,g_{BB^*\pi}}{M_{B^*}}\,\frac{\phi_{B^*}(x/\zeta)}{M^2_{B^*} - \zeta\,M_B^2}\,. \end{aligned} \tag{22}$$

The valence Fock state of the B^{*-} resonance consists of a b and a $\overline{\mathrm{u}}$ quark with an associated wave function similar to Eq. (16). Since the transverse parton momenta, defined with respect to the B^* momentum q, are integrated over, only the B^* distribution amplitude, $\phi_{B^*}(y)$, remains for which one may, for instance, apply the same ansatz as for the B meson. Its explicit form is irrelevant for the transition form factors as we will see below. The argument of the B^* distribution amplitude, x/ζ, equals the momentum fraction k_+/q_+ the b-quark carries w.r.t. the B^* meson. In the numerical analysis to be discussed below we take $f_{B^*}\;g_{BB^*\pi} = 20\,f_B$ which is compatible with a recent QCD sum rule analysis [10]. The coupling constant of the $BB^*\pi$ vertex is related to a parameter g in an effective Lagrangian in which the chiral and heavy quark symmetries are built in [9], by $g_{BB^*\pi} = 2M_B\,g/f_\pi(1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/M_B))$. From this it follows that for $q^2 \simeq q^2_{\mathrm{max}}$ the SPD $\widetilde{\mathcal{F}}^{(1)}_{\zeta\,\mathrm{res}}$ scales as $f_B\,g\,M_B/M_\pi \propto M_B^{1/2}$. This scaling law is in accordance with the one for the corresponding $B \to \pi$ form factors which has been found from the heavy quark limit of QCD in a model-independent way [8].

Putting all this together we obtain the numerical results for the b-u SPD $\widetilde{\mathcal{F}}^{(1)}_{\zeta}$ displayed in Fig. 2. Due to the characteristic features of the B-meson distribution amplitude, the overlap contribution (19) to $\widetilde{\mathcal{F}}^{1}_{\zeta}$ exhibits a bump at $x \simeq x_0$ provided ζ is smaller than x_0. That bump becomes more pronounced if ζ approaches x_0. The resonance contribution (22) provides the ridge at $x \simeq x_0\,\zeta$ where the B^* distribution amplitude is large. The resonance

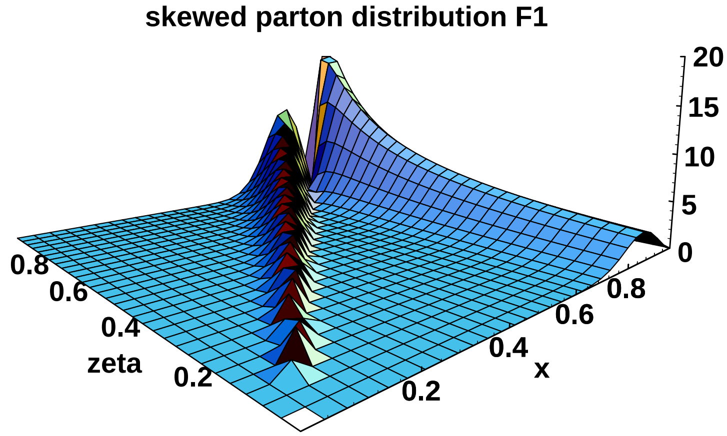

Figure 2: The SPD $\widetilde{\mathcal{F}}_\zeta^{(1)}(x)$ vs. x and ζ.

contribution generates a similar ridge in $\widetilde{\mathcal{F}}_\zeta^{(2)}$ while the overlap contribution to it is zero in our model.

Comparing the properties of both, the overlap and the resonance parts of which our SPDs consist, we see that they are continuous at the border points $x = 0$ and $x = \zeta$ while their derivatives do not exist there. Hence, our $\widetilde{\mathcal{F}}_\zeta^{(i)}(x)$ are non-analytic at the border points, a property that, according to Radyushkin [16], the SPDs should possess.

4 $B \to \pi$ form factors

While the form factor decomposition (2) is appropriate for the investigation of the SPDs, the form factors F_+ and F_0 are more suitable in applications to decay processes. We therefore refrain from discussing the form factors $F^{(i)}$, $i = 1, 2$ and present numerical results for $F_{+,0}$ only. The overlap contributions to the latter form factors are obtained from Eqs. (19), (10) and (12) by numerical integration and insertion of the resulting form factor $F^{(1)}$ into Eq. (3). The resonance contribution can be found along the same lines. In this case the x integration is trivial since it only applies to the B^* distribution amplitude and, as a change of variables reveals, this integral is just the normalisation (18). Hence, one obtains for the resonance contribution to the form factor F_+

$$F_{+,\text{res}}(q^2) \quad = \quad \frac{1}{2}\,\frac{q^2}{M_B^2}\,\frac{f_{B^*}\,g_{BB^*\pi}M_{B^*}}{M_{B^*}^2 - q^2}\,. \tag{23}$$

Note that the standard monopole term (see e.g. [9]) is modified by the factor q^2/M_B^2 which implies a q^2-dependent B^* coupling to the $B\pi$ system. That factor arises from our ansatz (22) in combination with Eq. (10) and (12). Since we consider a large range of momentum transfer (with respect to the meson radii) the appearance of such q^2 dependence is not

unreasonable. It forces the resonance contribution to vanish at $q^2 = 0$ in concord with the physical interpretation of the SPDs, see Eq. (10). At $q^2 \simeq q^2_{\rm max}$, on the other hand, the resonance term (23) is very close to the standard monopole term.
Analogously, one finds for the resonance contribution to the form factor F_0

$$F_{0,\rm res}(q^2) \;=\; \frac{q^2}{M_B^2}\left(1-\frac{q^2}{M_B^2}\right)\frac{f_{B^*}\,g_{B^*B\pi}}{2\,M_{B^*}}\,. \qquad (24)$$

A pre-factor, arising from the combination of Eqs. (3) and (22), cancels the B^*-pole in F_0. (We remind the reader of the fact that F_0 refers to a scalar current.)
In addition to the overlap and resonance contributions the form factors also receive contributions from perturbative QCD where a hard gluon with a virtuality of the order of M_B^2 is exchanged between the struck and the spectator quark. In Ref. [7] the perturbative contributions have been evaluated at large recoil within the modified perturbative approach in which the transverse degrees of freedom are retained and Sudakov suppressions taken into account. Since in Ref. [7] the same soft wave functions with Gaussian suppressions of large intrinsic transverse quark momenta have been applied as here (see Eqs. (14,16)), we can make use of the results presented in [7] and add them to our form factor predictions. At small recoil, $q^2 \geq 18$ GeV^2, the predictions for the perturbative contributions cease to be reliable because of the small virtualities some of the internal off-shell quarks and gluons acquire in this region.

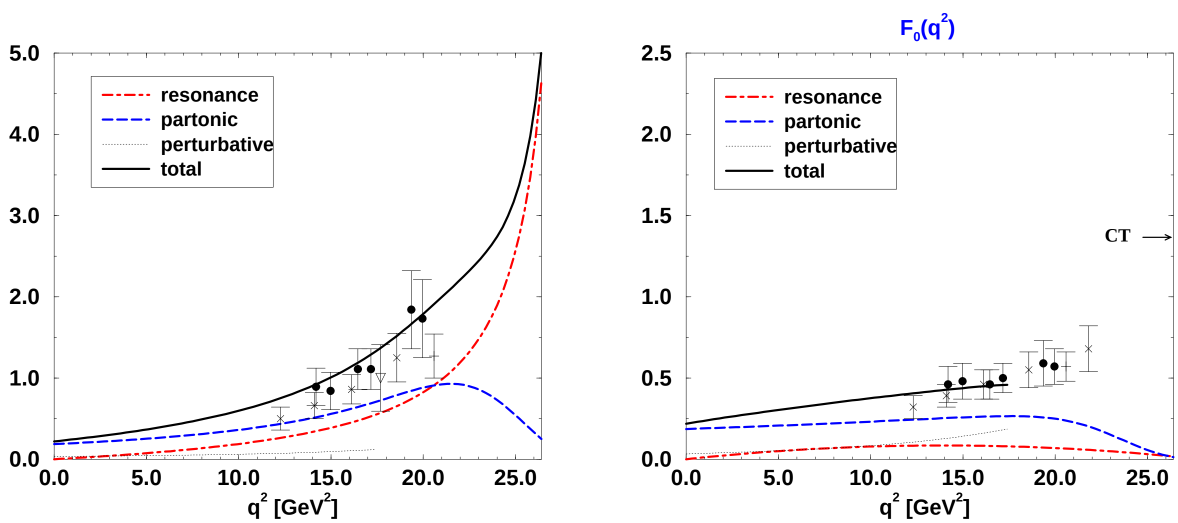

Figure 3: The form factors $F_+(q^2)$ and $F_0(q^2)$ vs. momentum transfer. Our predictions (solid lines) for the form factors are decomposed into resonance, overlap and perturbative contributions. The lattice QCD data, taken from Ref. [26], are shown for comparison. CT indicates the Callan-Treiman value, see text.

Numerical results for the three contributions, the overlap, the resonance and the perturbative one, are plotted in Fig. 3. In the case of the form factor F_+ we observe the dominance

of the overlap contribution at large recoil while the resonance contribution takes the lead at small recoil. This feature is expected to hold from the decomposition (10). The perturbative contribution, taken from Ref. [7], provides only a small correction to F_+, of the order of 10%, at large recoil and can be neglected at $q^2 \simeq q^2_{\rm max}$ as compared to the large resonance contribution. Actually, for the numerical analysis the perturbative contribution to F_+ is smoothly continued to zero for $q^2 \geq 18$ GeV^2. The sum of the three contributions to F_+ is in fair agreement with the lattice QCD results presented in [26].
Due to the absence of the B^* pole the form factor F_0 behaves differently; it is rather flat over the full range of momentum transfer. The perturbative contribution makes up a substantial fraction of the total result for F_0 at intermediate momentum transfer. Since, as we mentioned above, it becomes unreliable for $q^2 \gtrsim 18\,{\rm GeV}^2$ we are not in the position to predict F_0 at large q^2. A calculation of F_0 in that region would also require a detailed investigation of the scalar $B\pi$ resonances of which not much is known at present. Despite of this drawback our results for this form factor are also in fair agreement with the lattice QCD results [26] and, in tendency, seem to extrapolate to the B-sector analogue of the Callan-Treiman value

$$F_0^{CT}(q^2 = q^2_{\rm max}) \;=\; \frac{f_B}{f_\pi} + \mathcal{O}(M_\pi^2/M_B^2) \qquad (25)$$

which is provided by current algebra in the chiral limit [9].
In Fig. 4 we compare our results to a few other predictions of the $B \to \pi$ form factor F_+. We first mention the work by Bauer, Stech and Wirbel [4] in which the form factor has been calculated from a light-cone wave function overlap at $q^2 = 0$ and the result is used as a normalisation of a pole term. The BSW model has been applied to exclusive D- and B-meson decays and works quite well phenomenologically in many cases. Bauer, Stech and Wirbel employ a parameterisation of the pion wave function which resembles that of their B wave function (see (16), (17)) and, in contrast to us, normalise the pion wave function to unity. Doing so they find a larger overlap and, hence, a larger form factor as we do, see Fig. 4. At intermediated momentum transfer, on the other hand, our predictions for F_+ exceed the BSW result as a consequence of the superposition of resonance and overlap contribution.
Khodjamirian and Rückl [10] employed QCD light-cone sum rules for the calculation of the $B \to \pi$ form factors. In this approach the soft matrix elements are expressed as a series of collinear terms arising from operators of increasing twist; actually operators are used up to twist 4. The soft contributions are supplemented by α_s corrections to the twist-2 contribution and, at large momentum transfer where the QCD sum rules become unstable, by the B^* resonance matched to the sum of the other contributions at $q^2 \simeq 16\,{\rm GeV}^2$. As Fig. 4 reveals there are similar deviations between our predictions and those presented in [10] although to a lesser extend as in the case of the BSW model. A QCD sum rule analysis of the $B \to \pi$ form factors has also been attempted by Ball [11]. Although the results for F_+ presented in [10] and [11] agree fairly well with each other in general, differences in details are to be noticed.
Of particular interest is the value of the form factor F_+ at zero momentum transfer. It

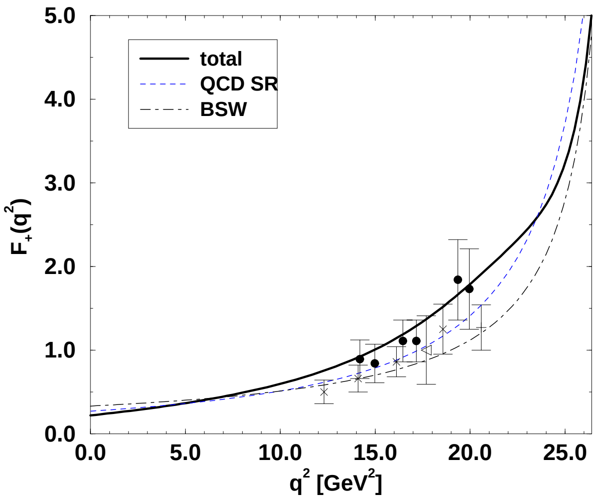

Figure 4: Comparison of various predictions for the form factor F_+. The solid line represents our result, the dashed one the QCD sum rule result of Ref. [10] and the dash-dotted one the BSW result [4]. The lattice QCD data are taken from Ref. [26].

plays an important role in the rates of the semi-leptonic B-meson decays and also in exclusive B-decays into $\pi\pi$ or other pairs of light pseudoscalar mesons. The widths for the latter processes are calculated on the basis of a (weak interaction) factorisation hypothesis with eventual QCD corrections. The quality of that hypothesis is not well-known. The factorising contribution to the decay amplitude is proportional to the $B \to \pi$ form factor F_0 at $q^2 \simeq 0$. From recent investigations of exclusive non-leptonic B-decays [27] we learned that a value of $F_0(0) = F_+(0)$ in the range of 0.30-0.33 is needed in order to account for the experimental decay widths [28] within that approach. Such a large value cannot easily be accommodated by the models. With the exception of the BSW model [4] where a value of 0.33 for $F_+(0)$ has been obtained, most of the other approaches, e.g. [5, 10, 11], provide values within the range of 0.2-0.3 which are often subject to substantial uncertainties so that there is no obvious conflict with the present theoretical understanding of non-leptonic B decays. For instance, in the QCD sum rule approach proposed in Ref. [10] a value of 0.27 has been found with an estimated error of about 0.05. We predict a value of 0.22 for $F_+(0)$ of which an amount of 0.03 originates from the perturbative contribution. For an assessment of the theoretical uncertainties of our results we have to consider the following items:

i) The overlap contribution is subject to Sudakov suppressions of the end-point region, $x \to 1$. Since the wave functions we are using, (14), (16) and (17), already suppress that region substantially (as compared, for instance, to the ones used in Ref. [5]) we do not expect the inclusion of the Sudakov factor to lead to dramatic effects. Moreover, the Sudakov suppressions are compensated to some extend by $\mathcal{O}(\alpha_s^2)$ corrections to the perturbative contributions [5]. Thus, we estimate that the net effect of Sudakov suppression

and $\mathcal{O}(\alpha_s^2)$ corrections does not exceed 10% of the overlap contribution to the form factor F_+.
ii) In the QCD sum rule approach [10, 11] a not unimportant contribution to the form factors comes from a two-particle twist-3 distribution amplitude. That distribution amplitude is constrained by the vacuum-pion matrix element of the pseudoscalar current being related to the divergence of the corresponding axial-vector current matrix element and known to acquire the large value $f_\pi M_\pi^2/(m_\mathrm{u}+m_\mathrm{d})$ where the m_q represent current quark masses. The implementation of this constraint into the light-cone wave function approach is somewhat ambiguous, and we therefore refrain from it in this article. It requires the introduction of a pion valence wave function component where quark and antiquark are in opposite helicity states. Such a component has been discussed in connection with the Melosh transform, see e.g. [29]. By examining several plausible parametrisations of this wave function component we find that its numerical impact on the overlap is around 10%.
iii) One may consider deviations of the pion distribution amplitude from the asymptotic form (15). Markedly broader distribution amplitudes, used for instance in recent QCD sum rule analyses [10, 11], clearly enhance the overlap with the B-meson wave function. On the other hand, they are in conflict with the $\pi\gamma$ transition form factor and the parton distributions of the pion [19]. In order to examine the bearing of the form of the pion distribution amplitude on the size of the overlap contribution we allow for a value of ± 0.01 for the second coefficient, B_2, in the Gegenbauer expansion of that distribution amplitude. Such a value of B_2, being still tolerated by the $\pi\gamma$ transition form factor within the light-cone wave function approach, leads to a change of ± 0.03 for $F_+(0)$.
iv) The uncertainty of the resonance contribution is proportional to that of the product of coupling constant and the B^* decay constant which is about 20%.
Combining these uncertainties with those arising from the input parameters in our approach (f_B, m_b) and the neglected order $\bar{\Lambda}/M_B$ corrections, we estimate the total uncertainty of our results for the $B \to \pi$ transition form factors to be about 20-25%.
Mannel and Postler [13] derived model-independent bounds for the $B \to \pi$ transition form factors from analyticity and unitarity. Inclusion of the values of the form factors and their derivatives at minimum and/or maximum momentum tighten the bounds considerably which then become a stringent test of the internal consistency of a model and its compatibility with QCD. We submit our form factor F_+ to this examination and take its values at $q^2 = 0$ and $q^2 = q^2_{\mathrm{max}}$ as well as its first two derivatives at $q^2 = 0$ as input. The result is plotted in Fig. 5a) and b). We observe that our prediction for F_+ lies comfortably within the bounds.
One may also consider bounds for given slope and curvature of F_+ at $q^2 = q^2_{\mathrm{max}}$. However, in contrast to the value of $F_+(q^2_{\mathrm{max}})$ itself which is dominated by the B^* pole, the higher derivatives of F_+ at small recoil may be sensitive to corrections from additional resonances, the treatment of perturbative corrections in that region etc. Nevertheless, for the sake of completeness, we plot the unitarity bounds with given $F'_+(q^2_{\mathrm{max}})$ and $F''_+(q^2_{\mathrm{max}})$ in Fig. 5c) and d). A mild violation of the bounds is observed. In view of the systematic and parametric uncertainties discussed above this is not to be considered as an inconsistency of our approach.

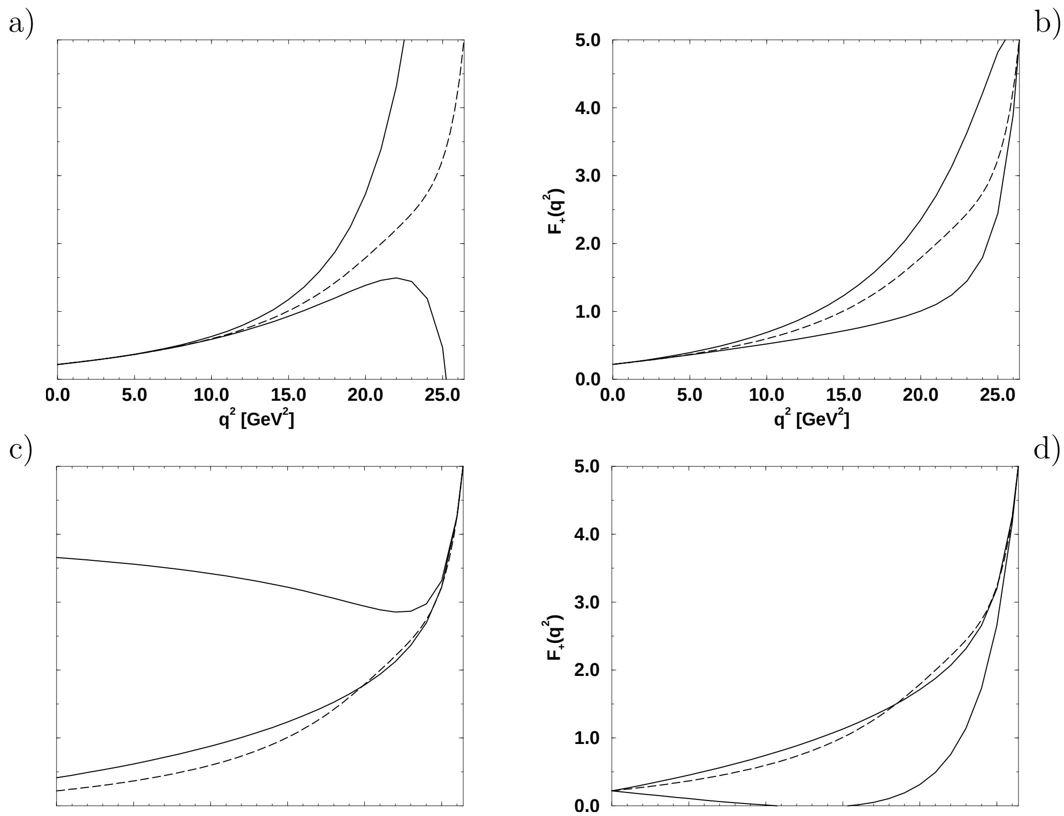

Figure 5: Testing unitarity. Solid lines: unitarity bounds [13]; dashed lines: our results for F_+. a) The value, slope and curvature of F_+ at $q^2 = 0$ are given; b) value and slope at $q^2 = 0$, value at $q^2_{\rm max}$; c) value, slope and curvature at $q^2_{\rm max}$; d) value at $q^2 = 0$, value and slope at $q^2_{\rm max}$.

Let us now turn to the discussion of the semi-leptonic decay rates $\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell$. The differential decay rate is given by

$$\begin{aligned}\frac{d\Gamma}{dq^2} &= \frac{G^2|V_{\rm ub}|^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_\pi^2 - M_\pi^2}}{q^4 M_B^2} \\ &\times \left\{ (1 + \frac{m_l^2}{2q^2})\, M_B^2 (E_\pi^2 - M_\pi^2)\, |F_+(q^2)|^2 + \frac{3m_l^2}{8q^2}\, (M_B^2 - M_\pi^2)^2\, |F_0(q^2)|^2 \right\}, \quad (26)\end{aligned}$$

where $E_\pi = (M_B^2 + M_\pi^2 - q^2)/(2M_B)$ is the pion energy in the B-meson rest frame. It is important to realize that for light leptons the scalar form factor F_0 plays a negligible role in the decay rate since its contribution appears with the square of the lepton mass, m_l. Therefore, the differential decay rates for the light-lepton modes determine $|V_{\rm ub} F_+(q^2)|$. On the other hand, the heavy-lepton decay mode $\bar{B}^0 \to \pi^+ \tau \bar{\nu}_\tau$ offers the possibility of exploring the scalar form factor.
Our predictions for the semi-leptonic decay rates into light or τ leptons are shown in

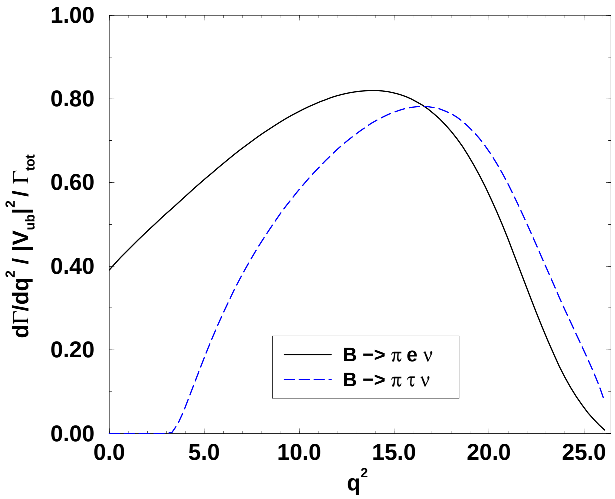

Figure 6: Predictions for the semi-leptonic differential decay widths, divided by $|V_{\rm ub}|^2$ and the total $\bar{B}^0$-meson width, $\Gamma_{\rm tot}$, vs. momentum transfer. Solid line: $\bar{B}^0 \to \pi^+ e\bar{\nu}_e$; dashed line: $\bar{B}^0 \to \pi^+ \tau\bar{\nu}_\tau$.

Fig. 6. For the scalar form factor F_0, which becomes important in the τ mode, we use her a simple, smooth interpolation between the CT value at $q^2 = q^2_{\rm max}$ and our results for F_0 below $q^2 = 18\,{\rm GeV}^2$. For the branching ratio of the light-lepton modes we find

$$BR[\bar{B}^0 \to \pi^+ e\bar{\nu}_e] \simeq BR[\bar{B}^0 \to \pi^+ \mu\bar{\nu}_\mu] = 1.9 \cdot 10^{-4} \cdot \left(\frac{|V_{\rm ub}|}{0.0035}\right)^2 .$$

The theoretical uncertainty of this prediction, dominated by that of the overlap contribution, amounts to about 30%. Our result is to be compared with the CLEO measurement [1]: $(1.8 \pm 0.4 \pm 0.3 \pm 0.2) \cdot 10^{-4}$ where the quoted errors refer to the statistical and systematical uncertainties and to the model dependence of the CLEO analysis, respectively.
For the τ channel we obtain

$$BR[\bar{B}^0 \to \pi^+ \tau\bar{\nu}_\tau] = 1.5 \cdot 10^{-4} \cdot \left(\frac{|V_{\rm ub}|}{0.0035}\right)^2 .$$

The estimated theoretical error amounts to about 30%. The ratio of both the branching ratios, in which the CKM matrix element cancels, amounts to 0.78 with an uncertainty of 15%.

5 Conclusions

We investigated the b-u SPDs within a light-cone wave function approach. Besides the usual overlap of the B and π valence Fock state wave functions we also considered higher

Fock states as well as annihilation contributions from non-diagonal overlaps and showed that these contributions provide only small, negligible corrections to the leading valence term. The B^* resonance is an important and, at small recoil, dominant contribution and has to be taken into account for a complete description of the transition form factors. The chief advantage of the SPD approach is that the skewedness parameter clearly separates the overlap from the resonance contribution and both the contributions can be added in an unambiguous way. From the b-u SPDs we calculated the $B \to \pi$ transition form factors by means of reduction formulas. Taking into account the corrections from perturbative physics [7], we obtain a reliable predication of F_+ for the entire range of momentum transfer and for F_0 up to about $18\,\mathrm{GeV}^2$. In particular, we obtain a value of 0.22 ± 0.05 for the form factors at maximum recoil. This value appears to be somewhat small if contrasted to the value required in $B \to \pi\pi$ decays (if the latter process is analysed on the basis of the factorisation hypothesis) but it is within range of other theoretical predictions of $F_+(0)$ [4, 5, 10, 11]. Generally, our results for the form factors are in fair agreement with the QCD sum rule result of Khodjamirian and Rückl [10] which is, in spirit, very close to the light-cone wave function approach. Our results are in agreement with lattice QCD data [26] and respect the unitarity bounds derived in Ref. [13], leaving aside mild violations for cases where the derivatives of F_+ at $q^2 = q^2_{\mathrm{max}}$ are used as input.
Using our form factors we calculated the differential and total decay rates for semi-leptonic $B \to \pi$ decays. Our predictions for the total decay for the process $\bar{B}^0 \to \pi^+ e \bar{\nu}_e$ is in good agreement with the recent CLEO measurement [1] if a value of 0.0035 is used for the CKM matrix element $|V_{\mathrm{ub}}|$. We stress that the knowledge of $F_+(0)$ is not sufficient for a prediction of the total decay rates since the q^2 dependence of the form factors is model-dependent.
We finally note that our approach can straightforwardly be applied to other heavy-to-light meson transition form factors. At small recoil the heavy quark symmetries [9] may turn out helpful in fixing parameters.

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